

Propellant-free Spacecraft Relative Maneuvering via Atmospheric Differential Drag

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found to be:



Current Problems in Space Industry to be Addressed

- Cost of carrying fuel is high
- Space real estate is limited and valuable
- Satellite detection is performed using reflection and/or heat emissions



Current Spacecraft Maneuvering with Propulsion Systems



mage credit to University of Surrey

Adaptive Lyapunov Control Strategy

A linear reference model is found by stabilizing the Schweighart and Sedwick model using a LQR, yielding the following:

$\dot{\boldsymbol{x}}_{d} = \underline{\boldsymbol{A}}_{d} \boldsymbol{x}_{d}, \quad \underline{\boldsymbol{A}}_{d} = \underline{\boldsymbol{A}} - \underline{\boldsymbol{B}} \underline{\boldsymbol{K}}, \quad \boldsymbol{x}_{d} = \begin{bmatrix} \boldsymbol{x}_{d} & \boldsymbol{y}_{d} & \dot{\boldsymbol{x}}_{d} & \dot{\boldsymbol{y}}_{d} \end{bmatrix}^{T}$

A Lyapunov function of the tracking error and its time derivative are found to be:

 $V = e^T \underline{P} e, \qquad e = x - x_d, \qquad \underline{P} \succ 0, \qquad \dot{V} = e^T (\underline{A}_d^T \underline{P} + \underline{P} \underline{A}_d) e + 2e^T \underline{P} (f(x) - \underline{A}_d x + \underline{B} a_{Drel} \hat{u} - \underline{B} u_d)$

If the desired guidance is a constant zero state vector (controller acts as a regulator) then the time derivative simplifies to:

$$\dot{V} = 2(\beta \hat{u} - \delta), \quad \beta = e^T \underline{P} \underline{B} a_{Drel}, \quad \delta = -e^T \underline{P} f(x), \quad \hat{u} = \begin{cases} 0 \\ 0 \end{cases}$$

Selecting: $\hat{u} = -sign(\beta) = -sign(e^T \underline{P}\underline{B})$ ensures the time derivative to be as small as possible A critical value for the magnitude of the drag acceleration that ensures Lyapunov stability is found to be: $a_{Drel} \geq \frac{\delta}{|\boldsymbol{e}^T \boldsymbol{P} \boldsymbol{R}|} = \frac{-\boldsymbol{e}^T \boldsymbol{P} \boldsymbol{f}(\boldsymbol{x})}{|\boldsymbol{e}^T \boldsymbol{P} \boldsymbol{R}|}$

- Mission life limited by fuel
- This fuel is expensive: ~\$5000/lb to transport it to Low Earth Orbit (<600km)
- Excess heat, dangerously flammable
- Volume cost
- Detectable

Differential Drag Theory



$|e \underline{I}\underline{D}|$ $|e \underline{I}\underline{D}|$ Expressions for the partial derivatives of the critical value in terms of matrices A and Q are $\frac{\partial a_{Dcrit}}{\partial \boldsymbol{Q}} = \mathbf{T}_{3}^{-1} \left(\frac{\partial \underline{\boldsymbol{P}}}{\partial \boldsymbol{Q}} \right) \left[\underline{\mathbf{I}}_{4x4} \otimes \mathbf{T}_{1}^{-1} \left(\frac{\partial a_{Dcrit}}{\partial \underline{\boldsymbol{P}}} \right) \right], \quad \frac{\partial a_{Dcrit}}{\partial \underline{\boldsymbol{A}}_{d}} = \mathbf{T}_{3}^{-1} \left(\frac{\partial \underline{\boldsymbol{P}}}{\partial \underline{\boldsymbol{A}}_{d}} \right) \left[\underline{\mathbf{I}}_{4x4} \otimes \mathbf{T}_{1}^{-1} \left(\frac{\partial a_{Dcrit}}{\partial \underline{\boldsymbol{P}}} \right) \right]$

Using these derivatives Ad and Q are adapted as follows:

 $\frac{dA_{ij}}{dt} = \kappa_A \left[-sign(\frac{\partial a_{Dcrit}}{\partial A_{ij}})\delta_A \right], \quad \frac{dQ_{ij}}{dt} = \kappa_Q \left[-sign(\frac{\partial a_{Dcrit}}{\partial Q_{ij}})\delta_Q \right], \\ \kappa_A = \begin{cases} 1 \text{ if } \left| \frac{\partial a_{Dcrit}}{\partial A_{ij}} \right| > \left| \frac{\partial a_{Dcrit}}{\partial A_{kl}} \right| \text{ for } i, j \neq k, l \\ 0 \text{ else} \end{cases}, \\ \kappa_Q = \begin{cases} 1 \text{ if } \left| \frac{\partial a_{Dcrit}}{\partial Q_{ij}} \right| > \left| \frac{\partial a_{Dcrit}}{\partial Q_{kl}} \right| \text{ for } i, j \neq k, l \\ 0 \text{ else} \end{cases}$

Origami-based Design





Origami: